$$(6-a) * a = (6-a) + a - 6 = 0$$

$$\therefore \quad a * (6-a) = (6-a) * a = 0$$

Therefore, each element $a$ of the set is invertible with $6-a$, being its inverse.

**Q. 5. Let $f(x) = x + 7$ and $g(x) = x - 7, x \in R$. Find the following :** *(JAC, 2015)*

**(i) *fof* (7)** **(ii) *fog* (7)**
**(iii) *gof* (7)** **(iv) *gog* (7)**

**Solution :** $\because f(x) = x + 7$ and $g(x) = x - 7$

(i)
$$fof(x) = f(x+7) = (x+7)+7 = x + 14$$
$$\therefore \quad fof(7) = 7 + 14 = 21$$

(ii)
$$fog(x) = f(x-7) = (x-7)+7 = x$$
$$\therefore \quad fog(7) = 7$$

(iii)
$$gof(x) = g(x+7) = (x+7)-7 = x$$
$$\therefore \quad gof(7) = 7$$

(iv)
$$gog(x) = g(x-7) = (x-7)-7 = x - 14$$
$$\therefore \quad gog(7) = 7 - 14 = -7$$

# 2 INVERSE TRIGONOMETRIC

## IMPORTANT FORMULAE

**Inverse Trigonometric Function :** If $\sin\theta = x$, then $\sin^{-1} x = \theta$

**Remember :** $\sin^{-1} x \neq \frac{1}{\sin x}$; etc.

**Principal Value Branches of Inverse Trigonometric Functions with their Domains and Ranges**

| Function | Domain | Range |
|---|---|---|
| $y = \sin^{-1} x$ | $[-1, 1]$ | $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ |
| $y = \cos^{-1} x$ | $[-1, 1]$ | $(0, \pi)$ |
| $y = \tan^{-1} x$ | R | $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ |
| $y = \cot^{-1} x$ | R $(-\infty < x < \infty)$ | $(0, \pi)$ |
| $y = \sec^{-1} x$ | $R - (-1, 1)$ | $(0, \pi) - \left\{\frac{\pi}{2}\right\}$ |
| $y = \text{cosec}^{-1} x$ | $R - (-1, 1)$ | $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\} - \{0\}$ |

- **Formulae Related with Inverse Trigonometric Functions :**

$$\sin^{-1}(\sin\theta) = \theta$$
$$\cos^{-1}(\cos\theta) = \theta$$
$$\tan^{-1}(\tan\theta) = \theta$$
$$\cot^{-1}(\cot\theta) = \theta$$
$$\sec^{-1}(\sec\theta) = \theta$$
$$\text{cosec}^{-1}(\text{cosec}\,\theta) = \theta$$
$$\sin(\sin^{-1}\theta) = \theta$$
$$\cos(\cos^{-1}\theta) = \theta$$
$$\tan(\tan^{-1}\theta) = \theta$$
$$\cot(\cot^{-1}\theta) = \theta$$
$$\sec(\sec^{-1}\theta) = \theta$$
$$\text{cosec}(\text{cosec}^{-1}\theta) = \theta$$
$$\text{cosec}^{-1} x = \sin^{-1}\frac{1}{x}$$

$\sec^{-1} x = \cos$ [illegible]

$\cot^{-1} x = \tan$ [illegible]

$$\sin^{-1}(-x) = -\sin^{-1} x$$
$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$
$$\tan^{-1}(-x) = \pi - \tan^{-1} x$$
$$\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x$$
$$\sec^{-1}(-x) = \pi - \sec^{-1} x$$
$$\cot^{-1}(-x) = \pi - \cot^{-1} x$$
$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$
$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$
$$\sec^{-1} x + \text{cosec}^{-1} x = \frac{\pi}{2}$$
$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$$

$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left[\frac{x-y}{1 \ldots}\right]$ [illegible]

$$\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$$
$$\cot^{-1} x \pm \cot^{-1} y = \cot^{-1}\left[\frac{xy \mp 1}{y \pm x}\right]$$
$$\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left\{x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right\}$$
$$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left\{xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right\}$$
$$2\tan^{-1} x = \sin^{-1}\left[\frac{2x}{1+x^2}\right] = \cos^{-1}\left[\frac{1-x^2}{1+x^2}\right] = \tan^{-1}\left[\frac{2x}{1-x^2}\right]$$

$$2\sin^{-1}x = \sin^{-1}\left\{2x\sqrt{1-x^2}\right\}$$
$$2\cos^{-1}x = \cos^{-1}(2x^2-1)$$
$$3\sin^{-1}x = \sin^{-1}(3x-4x^3)$$
$$3\cos^{-1}x = \cos^{-1}(4x^3-3x)$$
$$3\tan^{-1}x = \tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$$
$$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$$
$$\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x}$$
$$\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}}$$

### ➠ Multiple Choice Questions

1. The value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

2. $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} =$ (*BSEB*, 2010)

(a) $\tan^{-1}\frac{44}{29}$ (b) $\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{4}$

3. $\tan^{-1}(\sqrt{3}) - \sec^{-1}\alpha$ or 2 (*BSEB*, 2011; *AI CBSE*, 2012)

(a) $\pi$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

4. If $\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sin^{-1}\left(\frac{2y}{1+y^2}\right) = 2\tan^{-1}a$, then $a$ is equal to :

(a) $\frac{x-y}{1+xy}$ (b) $\frac{y}{1+xy}$ (c) $\frac{y}{1-xy}$ (d) $\frac{x+y}{1-xy}$

5. If $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then $x$ is equal to :

(a) 0 (b) $0, -\frac{1}{2}$

(c) $0, \frac{1}{2}$ (d) none of these

6. $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$ is equal to :

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{-3\pi}{4}$

7. The value of $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}\right)$ is :

(a) $\frac{\pi}{2}$ (b) 1 (c) 0 (d) $\frac{1}{2}$

8. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) =$

(a) $\frac{7\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{6}$

9. The value of $\tan\left(2\tan^{-1}\frac{1}{5}\right)$ is :

(a) $\frac{12}{5}$ (b) $\frac{1}{5}$ (c) $\frac{5}{12}$ (d) $\frac{7}{12}$

10. If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{3}$ then the value of $\cos^{-1}x + \cos^{-1}y$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\pi$

11. The principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is : (*BSEB*, 2015)

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

12. $\tan^{-1}x =$ (*BSEB*, 2015)

(a) $\cot^{-1}x$ (b) $\frac{1}{\cot^{-1}x}$ (c) $\cot^{-1}\frac{1}{x}$ (d) $-\cot^{-1}x$

13. $\cos^{-1}\frac{1-x^2}{1-x^2} =$ (*BSEB*, 2015)

(a) $2\cos^{-1}x$ (b) $2\sin^{-1}x$ (c) $2\tan^{-1}x$ (d) $\cos^{-1}2x$

**Ans.** 1. (c), 2. (d), 3. (b), 4. (d), 5. (a), 6. (b), 7. (b), 8. (a), 9. (c), 10. (c), 11. (d), 12. (c), 13. (c)

### ➠ Very Short Answer Type Questions

**Q. 1. Prove that :**

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}.$$ (*JAC*, 2011)

**Solution :**

Let $\tan^{-1}x = \alpha$ and $\tan^{-1}y = \beta$

$\therefore$ $x = \tan\alpha$ and $y = \tan\beta$

Now $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \frac{x+y}{1-xy}$

$\therefore$ $\alpha + \beta = \tan^{-1}\frac{x+y}{1-xy}$

or $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

**Q. 2. Prove that :**

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$ (*JAC*, 2013; *USEB*, 14)

**Solution :**

Let $\sin^{-1}x = \theta$

then, $x = \sin\theta$

$\Rightarrow$ $x = \cos\left(\frac{\pi}{2} - \theta\right)$

$\Rightarrow$ $\frac{\pi}{2} - \theta = \cos^{-1}x$

$\Rightarrow$ $\frac{\pi}{2} - \sin^{-1}x = \cos^{-1}x$ $(\because \sin x = \theta)$

$\Rightarrow$ $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

**Q. 3. Write the principal value of $\cos^{-1}[\cos(680°)]$.** [*CBSE*, 2014 (*Comptt.*)]

**Solution :** $\cos^{-1}[\cos(680°)]$

$= \cos^{-1}[\cos(2 \times 360° - 40°)]$
$= \cos^{-1}(\cos 40°)$
$= 40°$

**Q. 4. Determine the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.** *(USEB, 2013)*

**Solution :** $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

$\left(\because \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \text{ and } -\frac{\pi}{2} \le \frac{\pi}{4} \le \frac{\pi}{2}\right)$

**Q. 5. Evaluate : $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$** [*CBSE*, 2013 (*Comptt.*)]

**Solution :** $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

$= \sin^{-1}\sin\left(\pi - \frac{3\pi}{5}\right)$

$= \sin^{-1}\sin\frac{2\pi}{5}$

$= \frac{2\pi}{5} \quad \left[\because \frac{2\pi}{5} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

**Q. 6. Write the principal value of $\cot^{-1}\left(\tan\frac{\pi}{7}\right)$.** *(BSEB, 2014)*

**Solution :** $\cot^{-1}\left(\tan\frac{\pi}{7}\right) = \cot^{-1}\left\{\cot\left(\frac{\pi}{2} - \frac{\pi}{7}\right)\right\}$

$= \frac{\pi}{2} - \frac{\pi}{7}$

$= \frac{5\pi}{14}$

**Q. 7. Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.** [*AI, CBSE*, 2014 (*Comptt.*)]

**Solution :**

$\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right] = \tan^{-1}\left(-\sin\frac{\pi}{2}\right)$

$= \tan^{-1}(-1)$
$= -\tan^{-1} 1$

$= -\frac{\pi}{4}$

**Q. 8. Write the principal value of $\cos\left[\frac{\pi}{2} + \sin^{-1}\left(\frac{1}{3}\right)\right]$.** *(BSER, 2013)*

**Solution :** $\cos\left[\frac{\pi}{2} + \sin^{-1}\left(\frac{1}{3}\right)\right] = -\sin\left[\sin^{-1}\left(\frac{1}{3}\right)\right]$

$= -\frac{1}{3}$

**Q. 9. Find the principal value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$.** *(AI, CBSE, 2013, BSER, 14)*

**Solution :**

$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3}) = \tan^{-1}\sqrt{3} - \{\pi - \cot^{-1}(\sqrt{3})\}$

$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right)$

$= \frac{\pi}{3} - \frac{5\pi}{6}$

$= \frac{2\pi - 5\pi}{6}$

$= \frac{-3\pi}{6}$

$= -\frac{\pi}{2}$

**Q. 10. Write the principal value of**

**$[\tan^{-1}(-\sqrt{3}) + \tan^{-1}(1)]$** [*CBSE*, 2013 (*Comptt.*)]

**Solution :**

$\tan^{-1}(-\sqrt{3}) + \tan^{-1}(1) = -\tan^{-1}(\sqrt{3}) + \tan^{-1}(1)$

$[\because \tan^{-1}(-x) = -\tan^{-1} x]$

$= -\frac{\pi}{3} + \frac{\pi}{4}$

$= -\frac{\pi}{12}$

**Q. 11. Write the principal value of**

**$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$** *(CBSE, 2013)*

**Solution :**

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) = \tan^{-1}(1) + \pi - \cos^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{4} + \pi - \frac{\pi}{3}$

$= \frac{3\pi + 12\pi - 4\pi}{12}$

$= \frac{11\pi}{12}$

**Q. 12. Write the value of $\tan\left(2\tan^{-1}\frac{1}{5}\right)$.** *(CBSE, 2013)*

**Solution :**

$$\tan\left(2\tan^{-1}\frac{1}{5}\right) = \tan\left[\tan^{-1}\left\{\frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2}\right\}\right]$$

$= \tan\left\{\tan^{-1}\left(\frac{5}{12}\right)\right\}$

$= \frac{5}{12}$

**Q. 13. Write the value of**

**$\tan^{-1}\left[2\sin\left\{2\cos^{-1}\frac{\sqrt{3}}{2}\right\}\right]$** *(AI, CBSE, 2013)*

**Solution :**

$\tan\left[2\sin\left\{2\cos^{-1}\frac{\sqrt{3}}{2}\right\}\right] = \tan^{-1}\left[2\sin\left(2.\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left(2\sin\frac{\pi}{3}\right)$

$= \tan^{-1}\left(2.\frac{\sqrt{3}}{2}\right)$

$$= \tan^{-1}\sqrt{3}$$
$$= \frac{\pi}{3}$$

**Q. 14. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, $xy < 1$, then write the value of $x + y + xy$.** *(CBSE, 2014)*

**Solution :**

$$\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$$
$$\Rightarrow \quad \tan^{-1}\frac{x+y}{1-xy} = \frac{\pi}{4}$$
$$\Rightarrow \quad \frac{x+y}{1-xy} = \tan\frac{\pi}{4}$$
$$\Rightarrow \quad \frac{x+y}{1-xy} = 1$$
$$\Rightarrow \quad x + y = 1 - xy$$
$$\Rightarrow \quad x + y + xy = 1$$

**Q. 15. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$, then find the value of $x$.** *(CBSE, 2014)*

**Solution :** $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$

$$\Rightarrow \quad \sin^{-1}\frac{1}{5} + \cos^{-1} x = \sin^{-1} 1$$
$$\Rightarrow \quad \sin^{-1}\frac{1}{5} + \cos^{-1} x = \frac{\pi}{2}$$
$$\Rightarrow \quad \cos^{-1} x = \frac{\pi}{2} - \sin^{-1}\frac{1}{5}$$
$$\Rightarrow \quad \cos^{-1} x = \cos^{-1}\frac{1}{5}$$
$$\Rightarrow \quad x = \frac{1}{5}$$

**Q. 16. Write the principal value of**

$$\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$$ *(CBSE Delhi, 2012)*

**Solution :**

$$\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right) = \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$$
$$= \frac{\pi}{3} + 2.\frac{\pi}{6}$$
$$= \frac{\pi}{3} + \frac{\pi}{3}$$
$$= \frac{2\pi}{3}$$

**Q. 17. Find the principal value of**

$$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2).$$ *(CBSE Delhi, 2012)*

**Solution :**

$$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \tan^{-1}(\sqrt{3}) - \{\pi - \sec^{-1}(2)\}$$
$$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right)$$
$$= \frac{2\pi}{3} - \pi$$
$$= -\frac{\pi}{3}$$

## Short Answer Type Questions

**Q. 1. Prove that :**

*(USEB, 2009, 13; BSEB, 2011)*

$$\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$$

**Solution :**

Let $\sin^{-1}\frac{3}{5} = \theta$ and $\sin^{-1}\frac{8}{17} = \phi$

$$\therefore \quad \sin\theta = \frac{3}{5} \text{ and } \sin\phi = \frac{8}{17}$$
$$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$
and $$\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\frac{64}{289}} = \sqrt{\frac{225}{289}}$$
$$= \frac{15}{17}$$

Now, $$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$
$$= \frac{4}{5}\times\frac{15}{17} + \frac{3}{5}\times\frac{8}{17}$$
$$= \frac{60}{85} + \frac{24}{85}$$
$$= \frac{84}{85}$$
$$\Rightarrow \quad \theta - \phi = \cos^{-1}\left(\frac{84}{85}\right)$$
$$\therefore \quad \sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$$ **Hence Proved**

**Q. 2. Prove that :** *(JAC, 2011)*

$$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$$

**Solution :**

$$\text{L.H.S.} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8}$$
$$= \left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5}\right) + \left(\tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8}\right)$$
$$= \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{3}\times\frac{1}{5}}\right) + \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}\right)$$
$$= \tan^{-1}\left(\frac{\frac{8}{15}}{\frac{14}{15}}\right) + \tan^{-1}\left(\frac{\frac{15}{56}}{\frac{55}{56}}\right)$$
$$= \tan^{-1}\frac{8}{14} + \tan^{-1}\frac{15}{55}$$
$$= \tan^{-1}\frac{4}{7} + \tan^{-1}\frac{3}{11}$$
$$= \tan^{-1}\frac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\times\frac{4}{11}} = \tan^{-1}\frac{\frac{65}{77}}{\frac{65}{77}}$$
$$= \tan^{-1}(1) = \frac{\pi}{4} = \text{R.H.S.}$$

**Q. 3. If $\tan^{-1}\left(\dfrac{x-1}{x-2}\right) + \tan^{-1}\left(\dfrac{x+1}{x+2}\right) = \dfrac{\pi}{4}$, then find the value of $x$.** (*USEB*, 2011)

**Solution :**

$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right) = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{\frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)}}{\frac{(x-2)(x+2)-(x-1)(x+1)}{(x-2)(x+2)}}\right] = \frac{\pi}{4}$$

$$\Rightarrow \quad \frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1} = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad \frac{2x^2-4}{-3} = \frac{1}{1}$$

$$\Rightarrow \quad 2x^2-4 = -3$$

$$\Rightarrow \quad x^2 = \frac{1}{2}$$

$$\therefore \quad x = \pm\frac{1}{\sqrt{2}}$$

**Q. 4. Write the function of $\tan^{-1}\left(\dfrac{\cos x - \sin x}{\cos x + \sin x}\right)$, $x < \pi$ in the simplest form.** (*BSEB*, 2014)

**Solution :**

$$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right) = \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-x\right)\right\}$$

$$= \frac{\pi}{4}-x$$

**Q. 5. Solve the following equation :**

$$\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right) \quad (\textit{AI CBSE}, 2013)$$

**Solution :**

$$\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

$$\Rightarrow \quad \cos(\tan^{-1}x) = \sin\left(\tan^{-1}\frac{4}{3}\right) \quad \ldots(1)$$

We know that

$$\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}}$$

$\therefore$ (1) given

$$\cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \sin\sin^{-1}\left\{\frac{\frac{4}{3}}{\sqrt{1+\left(\frac{4}{3}\right)^2}}\right\}$$

$$\Rightarrow \quad \frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{1}{1+x^2} = \frac{16}{25} \quad \text{(Squaring both sides)}$$

$$\Rightarrow \quad 16+16x^2 = 25$$

$$\Rightarrow \quad 16x^2 = 9$$

$$\Rightarrow \quad x^2 = \frac{9}{16}$$

$$\Rightarrow \quad x = \pm\frac{3}{4}$$

**Q. 6. Prove that :** [*CBSE*, 2012, 13 (*Comptt.*)]

$$\cos^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5} = \cos^{-1}\frac{33}{65} = \tan^{-1}\frac{56}{33}$$

**Solution :**

Let $\cos^{-1}\dfrac{12}{13} = x$ and $\cos^{-1}\dfrac{4}{5} = y$.

Then,

$$\cos x = \frac{12}{13} \text{ and } \cos y = \frac{4}{5}$$

$$\therefore \quad \sin x = \frac{5}{13} \text{ and } \sin y = \frac{3}{5}$$

$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$

$$\Rightarrow \quad \cos(x+y) = \frac{12}{13}\cdot\frac{4}{5} - \frac{5}{13}\cdot\frac{3}{5}$$

$$\Rightarrow \quad \cos(x+y) = \frac{33}{65}$$

$$\Rightarrow \quad x+y = \cos^{-1}\frac{33}{65}$$

$$\Rightarrow \quad \cos^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5} = \cos^{-1}\frac{33}{65}$$

$$= \tan^{-1}\left\{\frac{\sqrt{1-\left(\frac{33}{65}\right)^2}}{\frac{33}{65}}\right\}$$

$$\left(\because \cos^{-1}x = \tan^{-1}\frac{\sqrt{1-x^2}}{x}\right)$$

$$= \tan^{-1}\left(\frac{56}{33}\right)$$

**Q. 7. Solve for $x$ :**

**$\sin^{-1}(1-x) - 2\sin^{-1}x = \dfrac{\pi}{2}$** [*CBSE*, 2013 (*Comptt.*)]

**Solution :** $\sin^{-1}(1-x) - 2\sin^{-1}x = \dfrac{\pi}{2}$

$$\Rightarrow \quad \sin^{-1}(1-x) = \frac{\pi}{2}+2\sin^{-1}x$$

$$\Rightarrow \quad 1-x = \sin\left\{\frac{\pi}{2}+2\sin^{-1}x\right\}$$

$$\Rightarrow \quad 1-x = \cos(2\sin^{-1}x)$$

Put $x = \sin\theta$, then.

$$1-\sin\theta = \cos(2\sin^{-1}\sin\theta)$$

$$\Rightarrow \quad 1-\sin\theta = \cos 2\theta$$

$$\Rightarrow \quad 1-\sin\theta = 1-2\sin^2\theta$$

$$\Rightarrow \quad \sin\theta = 2\sin^2\theta$$

$$\Rightarrow \sin\theta(1-2\sin\theta) = 0$$

$\Rightarrow \quad \sin\theta = 0, \sin\theta = \frac{1}{2}$

$\Rightarrow \quad x = 0, x = \frac{1}{2}$

$\Rightarrow \quad x = 0, \frac{1}{2}$

But $x = \frac{1}{2}$ does not satisfy the given equation

$\therefore \quad x = 0$

**Q. 8. Prove that :** [*BSEB*, 2014; *CBSE*, 2013]

$$\tan\left\{\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2} + \frac{1}{2}\cos^{-1}\frac{1-x^2}{1+x^2}\right\} = \frac{2x}{1-x^2}$$

**Solution :**

$$\text{LHS} = \left\{\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2} + \frac{1}{2}\cos^{-1}\frac{1-x^2}{1+x^2}\right\}$$

$$= \tan\left\{\frac{1}{2}\sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta} + \frac{1}{2}\cos^{-1}\frac{1-\tan^2\theta}{1+\tan^2\theta}\right\}$$

(Putting $x = \tan\theta$)

$$= \tan\left\{\frac{1}{2}\sin^{-1}\sin 2\theta + \frac{1}{2}\cos^{-1}\cos 2\theta\right\}$$

$$= \tan\left\{\frac{1}{2}.2\theta + \frac{1}{2}.2\theta\right\}$$

$$= \tan(\theta + \theta)$$

$$= \tan 2\theta$$

$$= \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2x}{1-x^2}$$

$= \text{RHS}$

**Q. 9. Prove that :** (*BSER*, 2013)

$$\frac{1}{2}\tan^{-1}x = \cos^{-1}\left\{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}\right\}^{\frac{1}{2}}$$

**Solution :** $\text{RHS} = \cos^{-1}\left\{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}\right\}^{\frac{1}{2}}$

$$= \cos^{-1}\left\{\frac{1+\sqrt{1+\tan^2\theta}}{2\sqrt{1+\tan^2\theta}}\right\}^{\frac{1}{2}}$$

(Putting $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$)

$$= \cos^{-1}\left(\frac{1+\sec\theta}{2\sec\theta}\right)^{\frac{1}{2}}$$

$$= \cos^{-1}\left\{\frac{1+\frac{1}{\cos\theta}}{\frac{2}{\cos\theta}}\right\}^{\frac{1}{2}}$$

$$= \cos^{-1}\left(\frac{1+\cos\theta}{2}\right)^{\frac{1}{2}}$$

$$= \cos^{-1}\left(\frac{2\cos^2\theta\ 2}{2}\right)^{\frac{1}{2}}$$

$$= \cos^{-1}\left(\cos^2\frac{\theta}{2}\right)^{\frac{1}{2}}$$

$$= \cos^{-1}\left(\cos^2\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x = \text{LHS}$$

**Q. 10. If $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$, then find the value of $x$.** (*CBSE Delhi*, 2009; *BSEB, USEB*, 2014)

**Solution :**

$\because \quad \tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

$\Rightarrow \quad \tan^{-1}\left(\frac{2x+3x}{1-6x^2}\right) = \frac{\pi}{4}$

$\Rightarrow \quad \frac{5x}{1-6x^2} = 1$

$\Rightarrow \quad 6x^2 + 5x - 1 = 0$

$\Rightarrow \quad (x+1)(6x-1) = 0$

$\therefore \quad x = -1, \frac{1}{6}$

But $x = -1$ does not satisfy the given equation

$\therefore \quad x = \frac{1}{6}$

**Q. 11. Solve for $x$ :**

**$\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$** [*AI CBSE*, 2014 (*Comptt.*)]

**Solution :**

$\tan^{-1}x \ \ 2\cot^{-1}x = \frac{2\pi}{3}$

$\Rightarrow (\tan^{-1}x + \cot^{-1}x) + \cot^{-1}x = \frac{2\pi}{3}$

$\Rightarrow \quad \frac{\pi}{2} + \cot^{-1}x = \frac{2\pi}{3}$

$\Rightarrow \quad \cot^{-1}x = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}$

$\Rightarrow \quad x = \cot\frac{\pi}{6}$

$\therefore \quad x = \sqrt{3}$

**Q. 12. Prove that :**

**$2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{4} = \tan^{-1}\frac{32}{43}$** (*JAC*, 2013)

**Solution :** $\text{LHS} = 2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{4}$

$$= \tan^{-1}\frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2} + \tan^{-1}\frac{1}{4}$$

$$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{1}{4}$$

$$= \tan^{-1} \frac{\frac{5}{12} + \frac{1}{4}}{1 - \frac{5}{12} \cdot \frac{1}{4}}$$

$$= \tan^{-1} \frac{32}{43}$$

$$= \text{RHS}$$

**Q. 13. Prove that :** *(CBSE, 2014)*

$$2 \tan^{-1} \left(\frac{1}{5}\right) + \sec^{-1} \left(\frac{5\sqrt{2}}{7}\right) + 2 \tan^{-1} \left(\frac{1}{8}\right) = \frac{\pi}{4}$$

**Solution :**

$$\text{LHS} = 2 \tan^{-1} \left(\frac{1}{5}\right) + \sec^{-1} \left(\frac{5\sqrt{2}}{7}\right) + 2 \tan^{-1} \left(\frac{1}{8}\right)$$

$$= 2 \left\{\tan^{-1} \left(\frac{1}{5}\right) + \tan^{-1} \left(\frac{1}{8}\right)\right\} + \sec^{-1} \left(\frac{5\sqrt{2}}{7}\right)$$

$$= 2 \tan^{-1} \left\{\frac{\frac{1}{5} + \frac{1}{8}}{1 - \frac{1}{5} \cdot \frac{1}{8}}\right\} + \tan^{-1} \sqrt{\left(\frac{5\sqrt{2}}{7}\right)^2 - 1}$$

$$(\because \sec^{-1} x = \tan^{-1} \sqrt{x^2 - 1})$$

$$= 2 \tan^{-1} \left(\frac{13}{39}\right) + \tan^{-1} \sqrt{\frac{50}{49} - 1}$$

$$= 2 \tan^{-1} \left(\frac{1}{3}\right) + \tan^{-1} \left(\frac{1}{7}\right)$$

$$= \tan^{-1} \left\{\frac{2 \cdot \left(\frac{1}{3}\right)}{1 - \left(\frac{1}{3}\right)^2}\right\} + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \left\{\frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \cdot \frac{1}{7}}\right\}$$

$$= \tan^{-1} (1) = \frac{\pi}{4} = \text{RHS}$$

**Q. 14. Prove that :** *(CBSE Delhi, 2012)*

$$\tan^{-1} \left(\frac{\cos x}{1 + \sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, x \in \left(-\frac{\pi}{4}, \frac{\pi}{2}\right)$$

**Solution :**

$$\text{LHS} = \tan^{-1} \left(\frac{\cos x}{1 + \sin x}\right) = \tan^{-1} \left[\frac{\sin\left(\frac{\pi}{2} - x\right)}{1 + \cos\left(\frac{\pi}{2} - x\right)}\right]$$

$$= \tan^{-1} \left[\frac{\sin\left(\frac{\pi - 2x}{2}\right)}{1 + \cos\left(\frac{\pi - 2x}{2}\right)}\right]$$

$$= \tan^{-1} \left[\frac{2 \sin\left(\frac{\pi - 2x}{4}\right) \cos\left(\frac{\pi - 2x}{4}\right)}{2 \cos^2\left(\frac{\pi - 2x}{4}\right)}\right]$$

$$= \tan^{-1} \left[\tan\left(\frac{\pi - 2x}{4}\right)\right]$$

$$= \frac{\pi - 2x}{4} = \frac{\pi}{4} - \frac{x}{2} = \text{RHS}$$

**Q. 15. Prove that :** *(CBSE Delhi, 2012)*

$$\sin^{-1} \left(\frac{8}{17}\right) + \sin^{-1} \left(\frac{3}{5}\right) = \cos^{-1} \left(\frac{36}{85}\right)$$

**Solution :**

$$\text{LHS} = \sin^{-1} \left(\frac{8}{17}\right) + \sin^{-1} \left(\frac{3}{5}\right)$$

Now let $\sin^{-1} \left(\frac{8}{17}\right) = x$ and $\sin^{-1} \left(\frac{3}{5}\right) = y$

$\therefore$ $\sin x = \frac{8}{17}$ and $\sin y = \frac{3}{5}$

$\cos x = \frac{15}{17}$ and $\cos y = \frac{4}{5}$

We know that

$$\cos (x + y) = \cos x \cos y - \sin x \sin y$$

$$= \frac{15}{17} \times \frac{4}{5} - \frac{8}{17} \times \frac{3}{5}$$

$$= \frac{60}{85} - \frac{24}{85} = \frac{36}{85}$$

$$\therefore \quad x + y = \cos^{-1} \left(\frac{36}{85}\right)$$

Hence $\sin^{-1} \left(\frac{8}{17}\right) + \sin^{-1} \left(\frac{3}{5}\right) = \cos^{-1} \left(\frac{36}{85}\right)$

**Hence Proved**

**Q. 16. Prove that :**

$$\cos \left(\sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2}\right) = \frac{6}{5\sqrt{3}}$$ *(CBSE Delhi, 2012)*

**Solution :** $\text{LHS} = \cos \left(\sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2}\right)$

Let $\sin^{-1} \frac{3}{5} = x$ and $\cot^{-1} \frac{3}{2} = y$

Then, $\sin x = \frac{3}{5}$ and $\cot y = \frac{3}{2}$

$\therefore \cos x = \frac{4}{5}$ and $\sin y = \frac{2}{\sqrt{3}}$, $\cos y = \frac{3}{\sqrt{3}}$

Now, $\cos \left(\sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2}\right)$

$$= \cos (x + y)$$

$$= \cos x \cos y - \sin x \sin y$$

$$= \frac{4}{5} \times \frac{3}{\sqrt{13}} - \frac{3}{5} \times \frac{2}{\sqrt{13}}$$

$$= \frac{12}{5\sqrt{3}} - \frac{6}{5\sqrt{13}} = \frac{6}{5\sqrt{13}} = \text{RHS}$$

**Q. 17. Prove that :**

$$4 (\cot^{-1} 3 + \text{cosec}^{-1} \sqrt{5}) = \pi$$ *(BSEB, 2015)*

**Solution :** Let $\text{cosec}^{-1} \sqrt{5} = \alpha$

$\Rightarrow$ $\text{cosec}\, \alpha = \sqrt{5}$

$\therefore \quad \cot\alpha = \sqrt{\operatorname{cosec}^2\alpha - 1} = \sqrt{(\sqrt{5})^2 - 1} = 2$

$\Rightarrow \quad \alpha = \cot^{-1} 2$

$\therefore\ 4(\cot^{-1} 3 + \operatorname{cosec}^{-1} \sqrt{5})$

$= 4(\cot^{-1} 3 + \cot^{-1} 2)$

$= 4\left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}\right)$

$= 4\tan^{-1}\left(\dfrac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\cdot\frac{1}{2}}\right) = 4\tan^{-1}\left(\dfrac{\frac{5}{6}}{\frac{5}{6}}\right)$

$= 4\tan^{-1} 1 = 4\cdot\frac{\pi}{4}$

$= \pi$

**➠ Long Answer Type Questions**

**Q. 1. Show that : $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$.** *(AI CBSE, 2013)*

**Solution :** LHS $= \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)$

Put $\sin^{-1}\frac{3}{4} = \theta$

then, $\sin\theta = \frac{3}{4}$

$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta}$

$= \sqrt{1-\left(\frac{3}{4}\right)^2} = \frac{\sqrt{7}}{4}$

$\therefore \quad$ LHS $= \tan\left(\frac{\theta}{2}\right)$

$= \sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\dfrac{1-\frac{\sqrt{7}}{4}}{1+\frac{\sqrt{7}}{4}}}$

$= \sqrt{\dfrac{4-\sqrt{7}}{4+\sqrt{7}}} = \sqrt{\dfrac{4-\sqrt{7}}{4+\sqrt{7}}\cdot\dfrac{4-\sqrt{7}}{4-\sqrt{7}}}$

$= \sqrt{\dfrac{(4-\sqrt{7})^2}{16-7}}$

$= \dfrac{4-\sqrt{7}}{3} =$ RHS

**Q. 2. Prove that :** *(BSEB, 2013)*

$$\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right] = \frac{2b}{a}$$

**Solution :**

$\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right]$

$= \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$, where $\cos^{-1}\frac{a}{b} = \theta$

$= \dfrac{\tan\frac{\pi}{4}+\tan\frac{\theta}{2}}{1-\tan\frac{\pi}{4}\tan\frac{\theta}{2}} + \dfrac{\tan\frac{\pi}{4}-\tan\frac{\theta}{2}}{1+\tan\frac{\pi}{4}\tan\frac{\theta}{2}}$

$= \dfrac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} + \dfrac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$

$= \dfrac{\left(1+\tan\frac{\theta}{2}\right)^2+\left(1-\tan\frac{\theta}{2}\right)^2}{\left(1-\tan\frac{\theta}{2}\right)\left(1+\tan\frac{\theta}{2}\right)}$

$= \dfrac{2\left(1+\tan^2\frac{\theta}{2}\right)}{1-\tan^2\frac{\theta}{2}}$

$= \dfrac{2}{\left(\dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)}$

$= \dfrac{2}{\cos\theta} = \dfrac{2}{\left(\frac{a}{b}\right)} = \dfrac{2b}{a} =$ RHS

**Q. 3. If $\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{x+4}\right) = \frac{\pi}{4}$, then find the value of $x$.** *(AI CBSE, 2014)*

**Solution :**

$\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{x+4}\right) = \frac{\pi}{4}$

$\Rightarrow \quad \tan^{-1}\left[\dfrac{\frac{x-2}{x-4}+\frac{x+2}{x+4}}{1-\frac{x-2}{x-4}\cdot\frac{x+2}{x+4}}\right] = \frac{\pi}{4}$

$\Rightarrow \quad \dfrac{(x-2)(x+4)+(x+2)(x-4)}{(x-4)(x+4)-(x-2)(x+2)} = \tan\frac{\pi}{4}$

$\Rightarrow \quad \dfrac{(x^2+2x-8)+(x^2-2x-8)}{(x^2-16)-(x^2-4)} = 1$

$\Rightarrow \quad \dfrac{2x^2-16}{-12} = 1$

$\Rightarrow \quad 2x^2-16 = -12$

$\Rightarrow \quad 2x^2 = 16-12 = 4$

$\Rightarrow \quad x^2 = 2$

$\Rightarrow \quad x = \pm\sqrt{2}$

**Q. 4. Prove that :**

$$\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, -\frac{1}{\sqrt{2}} \le x \le 1$$

*(AI CBSE, 2014)*

**Solution :** LHS $= \tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$

Put $x = \cos 2\theta$.

then LHS $= \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right]$

$= \tan^{-1}\left[\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right]$

$= \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$

$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

$= \tan^{-1}\left\{\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right\}$

$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\theta\right)\right\}$

$= \frac{\pi}{4}-\theta$

$= \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$ $(\because x = \cos 2\theta)$

= RHS

**Q. 5. Prove that :**

**$\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$**

(*CBSE Delhi*, 2008)

**Solution :**

LHS $= \cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18$

$= \left(\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{18}\right)+\tan^{-1}\frac{1}{18}$

$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}\right)+\tan^{-1}\left(\frac{1}{18}\right)$

$= \tan^{-1}\left(\frac{15/56}{55/56}\right)+\tan^{-1}\left(\frac{1}{18}\right)$

$= \tan^{-1}\frac{3}{11}+\tan^{-1}\frac{1}{18}$

$= \tan^{-1}\left(\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}\times\frac{1}{18}}\right)$

$= \tan^{-1}\left(\frac{65/198}{195/198}\right)$

$= \tan^{-1}\left(\frac{1}{3}\right)$

$= \cot^{-1}(3)$ = RHS

**Q. 6. Prove that :**

**$\tan^{-1}\left(\frac{2}{3}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{17}{6}\right)$** (*USEB*, 2010)

**Solution :**

Given : $\tan^{-1}\left(\frac{2}{3}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{17}{6}\right)$

or $\tan^{-1}\left(\frac{17}{6}\right)-\tan^{-1}\left(\frac{2}{3}\right)=\sin^{-1}\left(\frac{3}{5}\right)$

LHS $= \tan^{-1}\left(\frac{17}{6}\right)-\tan^{-1}\left(\frac{2}{3}\right)$

$= \tan^{-1}\frac{\frac{17}{6}-\frac{2}{3}}{1+\frac{17}{6}\cdot\frac{2}{3}}$

$\left[\because \tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}\right]$

$= \tan^{-1}\frac{\frac{17-4}{6}}{1+\frac{17}{9}}$

$= \tan^{-1}\left(\frac{13}{6}\times\frac{9}{26}\right)$

$= \tan^{-1}\left(\frac{3}{4}\right)$

Let $\tan^{-1}\left(\frac{3}{4}\right)=\theta$ ...(i)

$\Rightarrow$ $\tan\theta = \frac{3}{4}$

$\therefore$ $\sin\theta = \frac{3}{5}$ or $\theta = \sin^{-1}\left(\frac{3}{5}\right)$

Putting the value of $\theta$ in equation (i),

$\tan^{-1}\left(\frac{3}{4}\right)=\sin^{-1}\left(\frac{3}{5}\right)$ = RHS

**Q. 7. Prove that :**

**$\cos^{-1}(x)+\cos^{-1}\left\{\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right\}=\frac{\pi}{3}$**

[*AI CBSE*, 2014 (*Comptt.*)]

**Solution :** LHS $= \cos^{-1}(x)+\cos^{-1}\left\{\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right\}$

Putting $x = \cos\theta$

LHS $= \cos^{-1}(\cos\theta)+\cos^{-1}\left\{\frac{1}{2}\cos\theta+\frac{\sqrt{3-3\cos^2\theta}}{2}\right\}$

$= \theta+\cos^{-1}\left\{\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta\right\}$

Put $\frac{1}{2} = r\cos\alpha$ and $\frac{\sqrt{3}}{2} = r\sin\alpha$

Squaring and adding, we get

$r^2 = \frac{1}{4}+\frac{3}{4} = 1 \Rightarrow r = 1$

dividing, we get

$$\tan\alpha = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \alpha = \frac{\pi}{3}$$

$$\therefore \quad \text{LHS} = \theta + \cos^{-1}\{r\cos\alpha\cos\theta + r\sin\alpha\sin\theta\}$$

$$= \theta + \cos^{-1}\{r\cos(\alpha-\theta)\}$$

$$= \theta + \cos^{-1}\left\{\cos\left(\frac{\pi}{3}-\theta\right)\right\}$$

$$= \theta + \frac{\pi}{3} - \theta$$

$$= \frac{\pi}{3} = \text{RHS}$$

**Q. 8. Solve for $x$ :**

$$\mathbf{2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\,x)}$$

[*CBSE*, 2014 (*Comptt.*)]

**Solution :** $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\,x)$

$$\Rightarrow \quad \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\,\text{cosec}\,x)$$

$$\Rightarrow \quad \frac{2\cos x}{1-\cos^2 x} = 2\,\text{cosec}\,x$$

$$\Rightarrow \quad \frac{\cos x}{\sin^2 x} = \frac{1}{\sin x}$$

$$\Rightarrow \quad \frac{\cos x}{\sin x} = 1$$

$$\Rightarrow \quad \cot x = 1 = \cot\frac{\pi}{4}$$

$$\therefore \quad x = n\pi + \frac{\pi}{4},\ n \in I$$

**Q. 9. Prove that :**

$$\mathbf{\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}}$$

(*CBSE Delhi*, 2008, 09 & *AI*, 10)

**Solution :** We have

$$\text{LHS} = \left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5}\right) + \left(\tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8}\right)$$

$$= \tan^{-1}\frac{\left(\frac{1}{3}+\frac{1}{5}\right)}{\left(1-\frac{1}{3}\times\frac{1}{5}\right)} + \tan^{-1}\frac{\left(\frac{1}{7}+\frac{1}{8}\right)}{\left(1-\frac{1}{7}\times\frac{1}{8}\right)}$$

$$= \tan^{-1}\frac{(8/15)}{(14/15)} + \tan^{-1}\frac{(15/56)}{(55/56)}$$

$$= \tan^{-1}\frac{8}{14} + \tan^{-1}\frac{15}{55}$$

$$= \tan^{-1}\frac{4}{7} + \tan^{-1}\frac{3}{11}$$

$$= \tan^{-1}\frac{\left(\frac{4}{7}+\frac{3}{11}\right)}{\left(1-\frac{4}{7}\times\frac{3}{11}\right)}$$

$$= \tan^{-1}\frac{(65/77)}{(65/77)}$$

$$= \tan^{-1} 1$$

$$= \frac{\pi}{4} = \text{RHS}$$

**Q. 10. Prove that :** (*JAC*, 2009; ***BSEB***, 2013)

$$\mathbf{2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}}$$

**Solution :**

$$\because \quad 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\therefore \quad 2\tan^{-1}\frac{1}{3} = \tan^{-1}\left(\frac{2\times\frac{1}{3}}{1-\frac{1}{9}}\right)$$

$$= \tan^{-1}\left(\frac{2/3}{8/9}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

Now

$$\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} = \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}}\right)$$

$$= \tan^{-1}\left(\frac{25/28}{25/28}\right)$$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$ **Proved.**

## NCERT QUESTIONS

**Q. 1. Write the value of** $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$.

(*CBSE*, 2008, 11; *JAC*, 14)

**Solution :**

$$\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right] = \sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right]$$

$$\left[\because \sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\frac{1}{2} = -\frac{\pi}{6}\right]$$

$$= \sin\frac{3\pi}{6} = \sin\frac{\pi}{2} = 1$$

**Q. 2. Prove that :**

$$\mathbf{\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right] = \frac{x}{2},\ x \in \left(0, \frac{\pi}{4}\right)}$$

[*CBSE*, 2011, 14; *AI CBSE*, 14 (*Comptt.*)]

**Solution :**

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right];\ x \in \left(0, \frac{\pi}{4}\right)$$

$$= \cot^{-1}\left\{\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right\}$$

$$\left[\because \left(\cos\frac{x}{2}\pm\sin\frac{x}{2}\right)^2 = \cos^2\frac{x}{2}+\sin^2\frac{x}{2} \pm 2\sin\frac{x}{2}\cos\frac{x}{2} = 1\pm\sin\frac{x}{2}\right]$$

$$= \cot^{-1}\left[\frac{\left|\cos\frac{x}{2}+\sin\frac{x}{2}\right|+\left|\cos\frac{x}{2}-\sin\frac{x}{2}\right|}{\left|\cos\frac{x}{2}+\sin\frac{x}{2}\right|-\left|\cos\frac{x}{2}-\sin\frac{x}{2}\right|}\right]$$

$$\left[\because \sqrt{x^2} = \sqrt{|x^2|} = |x|\right]$$

$$= \cot^{-1}\left\{\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right\}$$

$$\left[\because 0 < \frac{x}{2} < \frac{\pi}{8} \therefore \cos\frac{x}{2} > \sin\frac{x}{2}\right]$$

$$= \cot^{-1}\left\{\cot\frac{x}{2}\right\} = \frac{x}{2}$$

**Q. 3. Find the value of $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$.**

(*CBSE*, 2011)

**Solution :** $\tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{x-y}{x+y}\right)$

$\because \tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x-y}{1-xy}\right)$

$$\therefore \tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{x-y}{x+y}\right)$$

$$= \tan^{-1}\left[\frac{\frac{x}{y}+\frac{x-y}{x+y}}{1-\frac{x}{y}\left(\frac{x-y}{x+y}\right)}\right]$$

$$= \tan^{-1}\left[\frac{x^2+xy+xy-y^2}{xy+y^2-x^2+xy}\right]$$

$$= \tan^{-1}\left[\frac{x^2+2xy-y^2}{-x^2+2xy+y^2}\right]$$

**Q. 4. Write the value of $\tan^{-1}\left[\tan\frac{3\pi}{4}\right]$.**

(*CBSE*, 2011)

**Solution :**

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left\{\tan\left(\pi - \frac{\pi}{4}\right)\right\}$$

$$= \tan^{-1}\left\{\tan\left(-\frac{\pi}{4}\right)\right\}$$

$$= \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$$

$$= \tan^{-1}(-1) = -\tan^{-1}(1)$$

$$= -\frac{\pi}{4}$$

**Q. 5. Prove that :**

**$2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$** (*CBSE*, 2011)

**Solution :**

$$\text{LHS} = 2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$\because \quad 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$$

$$\therefore 2\tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\left(\frac{1}{1-\frac{1}{4}}\right) = \tan^{-1}\frac{4}{3}$$

$$\Rightarrow 2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$\therefore \quad \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right)$$

$$= \tan^{-1}\left(\frac{28+3}{17}\right)$$

$$= \tan^{-1}\left(\frac{31}{17}\right) = \text{RHS}$$

**Q. 6. Prove that :**

**$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$.**

**(*CBSE*, 2011, 13)**

**Solution :**

$$\text{LHS} = \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) \quad \text{...(i)}$$

We know that

$$\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$\therefore \quad \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{10}}\right)$$

$$= \tan^{-1}\left(\frac{7}{9}\right)$$

$\therefore$ From equation (i),

$$\text{LHS} = \tan^{-1}\left(\frac{7}{9}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{72}}\right)$$

$$= \tan^{-1}\left(\frac{56+9}{65}\right)$$

$$= \tan^{-1}\left(\frac{65}{65}\right)$$

$$= (1)$$

$$= \frac{\pi}{4} = \text{RHS}$$